Product Line Design with Frictions*

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Abstract

We study a monopolist's product line design problem with search frictions. Consumers only evaluate a random subset of products in the menu, limiting the monopolist's ability to perfectly match contracts to consumer types. This creates a tradeoff faced when expanding the product line between extracting more rents from different consumer types and increased matching costs. We show that when consumers are limited to seeing a single random product out of the menu, then the optimal menu for the monopolist always contains a single offer. When consumers observe more than one product, we show that a balanced menu where all products are seen by a consumer with the same probability is never optimal. The monopolist rather has an incentive to "bias" the menu so that some of the products are observed more often than others. Using an unbalanced menu has an impact on the quality provided to low valuation consumers, either reinforcing or reducing the distortions generated by asymmetric information. We discuss the consequences on quality provision, as well as the welfare effects of these distortions.

JEL codes: D82, D83, L15

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1 Introduction

Consumers are exposed to multiple products, brands, and prices. This makes the consumer's decision problem a complex process (Gilboa et al. (2021)), demanding both time and cognitive resources for the evaluation of these multiple alternatives. Recent empirical evidence (e.g., Sovinsky Goeree (2008), Honka and Chintagunta (2017), Honka et al. (2017), Honka et al. (2019),Abaluck and Adams-Prassl (2021), and Aguiar et al. (2023)) suggest that consumers often fail to consider all options during their purchase decisions, and that this behavior has consequences in both terms of estimation and policy evaluations.

In the other hand with the technology advancements used in marketplaces, sellers have gain access to the use of sophisticated algorithms and marketing strategies in the pursuit of obtaining higher profits. However, even in these sophisticated markets they not always have full control over those mechanisms either. Moreover, if consumers experiment choice overload or limited attention, then the success of more complex strategies could also be limited. These limitations then change the incentives for how sellers approach their business strategies and how they try to adapt also to consumer's behavior; not only in terms of the advertising and marketing strategies, but also in terms of the products they would ultimately want to offer

Our goal is to present a simple model that captures the basic trade-off between expanding the product line to extract more rents from different types of consumers and the increasing complexity that a larger product line involves. We consider a setting in which a seller has limited control over the matching between consumers and offers, while consumers face frictions in the interaction with the seller, and remain uninformed about the specifics of her product line.

We propose a framework to study how a monopolist will determine her product line when consumers are unaware of all the alternatives they have available. We do so by introducing search frictions in a canonical price discrimination setting à la Mussa and Rosen (1978). We model these search frictions as random samples obtained by the consumers about the products they have available. This provides a novel application of "sampling" (e.g., Dhangwatnotai et al. (2015), Fu et al. (2021)) for the consumer's problem, and allows us to study how within-firm search shapes the incentives of the firm.

In our model, a monopolist designs a menu of products contained in a unit-measure of slots in order to maximize her expected profits. Each product is characterized by their quality, price, and fraction of slots it uses. The fraction of slots a product uses determines how likely this product will enter consumers' consideration sets, i.e., how often it is sampled by consumers. Consumers have single unit demands, heterogeneous valuations for quality, and could inspect only a limited number of slots at random. We assume inspecting slots is costless but there is an exogenous sample size that determine the number of slots they are able to sample. In the baseline model, there is no heterogeneity in the number of slots consumers sample and all consumers have the same sample size which is known by the monopolist. Despite having a commnon sample size, the effective size of the consumers' consideration will vary across consumers as only some of them will end up inspecting slots containing different products. Therefore, consideration sets size and compositions are random in our setting.

We find that if consumers can inspect only a single slot, then the monopolist designs a menu with a single option. That is, when search frictions are extreme, the monopolist is better off removing all variety and uncertainty from the menu, focusing on producing only one version of his product for all consumers to evaluate. Instead, if frictions are less severe and consumers are able to inspect two slots, we show that the monopolist offers again a differentiated menu, but it is never optimal for her to offer a menu in which all products use the same proportion of slots, and the optimal menu is always *unbalanced*: the monopolist has an incentive to "bias" the distribution of slots, making some products more likely to enter the consumers' consideration sets than others. In turn, having an unbalanced menu would have an impact on the quality provided in the low quality product, either reinforcing or reducing the distortions genererated by the presence of asymmetric information only.

In the case of a single sample, the optimality of a single-product menu comes from having consumers being unable to compare two different products regardless of the distribution of slots. Therefore, no incentive compatibility constraint are relevant for the monopolist, and only participation constraints could be binding and determine the structure of the optimal products. This is no longer true once consumers can inspect more than one slot, as now there is a chance they would be able to compare two different offers if these are part of the menu. This results in having incentive compatibility constraints being relevant again, influencing the structure of the optimal menu. However, having a noisy match between products and consumers with different valuations changes the characteristics of the products that the optimal menu will contain, as either lower or bigger distortions on the quality provided to low types could arise in this case. In particular, since there is a possibility of having some consumers inspecting only slots containing the same product, there is an extra incentive to increase the sampling probability of the most profitable product, further distorting the composition of the optimal menu.

Related Literature

Our work is inspired by the literature in the intersection of economics and computer science studying complexity in mechanism design problems (e.g., Babaioff et al. (2018), Bergemann et al. (2021), Daskalakis and Zampetakis (2020), Dhangwatnotai et al. (2015), Fu et al. (2021), Hart and Nisan (2017, 2019)). This literature often studies problems in which the designer does not have a prior defined over an unknown characteristic of the environment, but can rely on samples to improve his designs. We depart from this literature by assuming agents (consumers) are uninformed about an attribute of the mechanism (the products in the menu) and they are the ones that must rely on samples to make their decisions instead of the designer (monopolist).

Our work contributes to the literature of product line design and adverse selection (e.g., Mussa and Rosen (1978)) by introducing search frictions. Recent interest in competitive models within this framework has grown (e.g., Garrett et al. (2018), Lester et al. (2019), and Fabra and Montero (2022)). However, these studies focus on search frictions occurring between firms, as a form of imperfect competition, while ignoring any frictions within the product lines of each firm. In contrast, our model explores the problem of a monopolist with search occurring within his own product line due to having consumers unaware of the composition of his product line. Nocke and Rey (2023) also studies a model with within-firm search but in a different setting where products characteristics are fixed, optimal pricing is uniform, and search is sequential. They focus on the different type of equilibria in terms of products positioning. In our model, both products characteristics (quality) and their prices are endogenous, search is simultaneous, and non-uniform pricing could be profitable.

Our model is also related to the literature combining information design into mechanism design problems (e.g., Krähmer (2020), Mensch (2022), Bergemann et al. (2022), Doval and Skreta (2022), and Cusumano et al. (2023)). However, in these models consumers must acquire information about their valuations but there is no uncertainty about the offerings of the seller or sellers. Instead, in our model consumers face no uncertainty about their valuations, and frictions come from the lack of information about the alternatives offered by the seller.

As we noted before, the consumers' behavior in our framework is motivated by complexity concerns. As in Safonov (2022), agents are boundedly rational and sample uniformly at random from an unobservable menu. While Safonov (2022) focuses on the complexity of the decision rule used by the agents, we instead focus on the design problem faced by the seller and take the behavior of the agents as fixed.

An alternative equilibrium concept using samples have been proposed previously by Osborne and Rubinstein (1998) and Osborne and Rubinstein (2003). Spiegler (2006) uses this equilibrium notion to study a similar setting to ours, albeit in a competitive environment where agents are aware of the alternatives they have but face uncertainty about the values those options would have for them. In our setting, agents remain unaware of the options they have available but face no uncertainty on the value those options have once revealed available. This makes our framework closer to the environment in Carroll (2015) which studies a moral hazard problem in which the principal must design an incentive scheme but is unaware of the set of actions the agent has available. However, in our model the agents (i.e., consumers) are the ones whose are unaware of options available instead of the principal.

Similar to Gerardi and Maestri (2020), Doval and Skreta (2022), Bergemann et al. (2022), and Sandmann (2023), the optimal menu in our setting could contain a single offer even in cases where the standard Mussa and Rosen (1978) solution involves offering a complete product line. Our contribution to this literature lies on providing search frictions as an alternative rational for the reduction on the product variety.

Our model is also closely related to classic models of price discrimination as Varian (1980) and Burdett and Judd (1983) in which consumers vary in the number of offers they observe. However, our framework includes asymmetric information and a single firm with an unknown menu instead of having multiple firms competing in an environment without taste heterogeneity. We also depart from this classic models by providing behavioral interpretation of this phenomenon.

Our paper is also related to the literature of limited consideration in economics and marketing (e.g., Eliaz and Spiegler (2011), Honka and Chintagunta (2017), Fershtman and Pavan (2022), Aguiar et al. (2023)). Our model features random consideration sets that are determined by the structure of the menu designed by the monopolist.

Outline

Section 2 describes the baseline model. Section 3 discusses some benchmarks. In Section 4 we characterize the optimal menu with a single sample. In Section 5 we discuss the case of two samples. Section 6 concludes.

2 Model

We consider the problem of a monopolist producing a vertically differentiated good interacting with a unit measure of consumers. Producing a good of quality $q \in [0, \overline{Q}]$ has per unit costs of $\varphi(q)$, where the cost function φ is twice continuously differentiable, strictly increasing, strictly convex, and satisfies $\varphi(0) = \varphi'(0) = 0$ and $\varphi'(Q) \geq \theta_h$.

There is a unit measure of *slots* available for the monopolist to position his products. Formally, a *product* or contract j is defined by a triple (q_j, p_j, x_j) where q_j and p_j are the quality and price of product j respectively, while $x_j \in (0, 1]$ denotes the fraction of slots used by product j. We also refer to x_j as the *salience* of product j. The salience of a product determines how likely is this product to enter consumers' consideration sets. Note that by assuming $x_j > 0$ we rule out the possibility of having products without salience ($x_j = 0$).[1] We will abuse notation and refer sometimes to a product also only by their quality and price, ignoring the associated salience.

A *menu* or mechanism M is defined as a finite collection of products such that

i. $\sum_{j \in M} x_j = 1$,

ii. $(q_i, p_i) \neq (q_j, p_j)$ for any $i, j \in M$ such that $i \neq j$, and

iii. $|M| \leq \overline{m}$,

where $\overline{m} \geq 2$ is the maximum number of products a menu could contain. Therefore, a menu must use all available slots, no two products can have identical quality and price, and there is a limit on the number of products a menu could contain.

There is a unit measure of consumers in the market. Consumers have single-unit demands and heterogeneous valuation for quality. A fraction μ_ℓ of consumers has low valuation ($\theta_\ell > 0$) per unit of quality, while a fraction $\mu_h = 1 - \mu_\ell$ has high valuation ($\theta_h > \theta_\ell$). If a consumer with valuation θ purchases a product of quality q and price p, she obtains utility

$$\theta q - p,$$

[1] This rules out the possibility of using zero salience decoys, or paper-launching products. This is not a concern in our model, but could be a desired feature by the seller in other settings.

while the monopolist per-unit profits are

$$p - \varphi(q).$$

Consumers are endowed with an exogenous sample size n that determines how many slots they are able to inspect. After examining n slots drawn uniformly at random, consumers must decide which product on these slots to purchase if any. Consumers in our model have a misspecified model about the menu offered by the monopolist: consumers believe that the complete menu offered by the monopolist correspond to the products in their sampled slots only. That is, they ignore that there are limitations on the number of products they observed and the possibility that other products could be available. As consumers browse among products of a single seller, cannot decide the intensity of their search, and have private information, our framework is a random within-firm search model with asymmetric information. This make our work related to the recent literature introducing Burdett and Judd (1983) search model into asymmetric information environments (e.g., Garrett et al. (2018) and Lester et al. (2019)).

We also assume that if a consumer is indifferent between two or more products, she breaks ties in favor of one of the products generating the highest profits for the monopolist among them. If the monopolist is indifferent between two or more menus, then he breaks ties in favor of the consumers, choosing the menu that gives higher utility to consumers.

Discussion

Frictions in our model are captured by the interaction of the slots available for the monopolist and the limited consideration (i.e., sample size) of consumers. These frictions allow a few different interpretations. The most direct comes in the form of search or informational frictions as in the original model random sample size model in Burdett and Judd (1983). Under this interpretation, a key difference is that in our model the probabilities are not completely exogenous, as the seller could influence them by adjusting his product line and how he uses the available slots.

Our model also has an alternative behavioral or bounded-rationality interpretation. Under this interpretation, consumers' limited attention or capacity to process information restrict the number of products they are able to evaluate. Moreover, the frictions in our model could also be interpreted as consumers holding a misspecified model about the monopolist design problem: they assume the complete product line offered by the seller is given only by the products they have observed. Since there is no way for consumers to learn more about the menu, nor for the seller to further inform them about the details of the menu, such a misspecified model cannot be corrected.

Finally, another suitable interpretation is with respect to advertising. We can think that the seller holds a fixed budget that must used to advertise their complete product line. Under this interpretation, slots represent the advertisement slots available, or how much of the budget is used in advertising each particular product. Using more slots makes it more likely that the product enters the consideration set of the potential buyers, but limits the number of slots or resources available to advertise the other products in the product line. These slots could also represent how focused is each ad in a particular product, how the products are organized in the shelves of a retail store, or how much of a website space is used for each product in a digital platform.[2]

3 Benchmarks

In this section we review the solution in the case of full consideration, that is, when consumers are able to evaluate all slots available. We start by characterizing the efficient or symmetric-information product line in which the products can be perfectly tailored to the preferences of each type of consumer. Then, we characterize the profit maximizing product line in the case of full consideration. This correspond to the solution in the original model of Mussa and Rosen (1978).

3.1 Efficient product line

We start with the efficient or symmetric-information case in which the monopolist could perfectly identify different type of consumers, offering personalized products to each one of them. That is, a setting in which both information and search frictions are absent.

Since the monopolist is able to identify the type of the consumer perfectly, he can capture the full value obtained by the consumer as revenue. Therefore, there is no trade off between maximizing the surplus and collecting revenue, and the monopolist will offer a products with the quality level that maximizes the surplus for each type of consumer, i.e., the provision of quality would be efficient.

[2]Nocke and Rey (2023) also considered a setting in which the seller could allocate different products in a fixed set of slots. However, in their setting there is no price discrimination in equilibrium as it is optimal to charge the same price for each product. Villas-Boas (2004) and Eliaz and Spiegler (2011) consider instead settings in which advertisement is costly but where the design dimension is further restricted.

In particular, for consumers with valuation θ_i, the surplus maximizing quality would be

$$q_i^* = \arg\max_q \theta_i q - \varphi(q).$$

This implies that the quality provided to each type of consumer is defined by the optimality condition

$$\varphi'(q_i^*) = \theta_i.$$

We denote by S_i^* for $i = \ell, h$ the surplus under the efficient quality provision for type θ_i, i.e.,

$$S_i^* = \theta_i q_i^* - \varphi(q_i^*).$$

Since there is symmetric information, the monopolist could charge a price that captures all the utility that a good of quality q_i^* generates for a consumer with valuation θ_i, i.e., $p = \theta_i q_i^*$. That is, the monopolist will offer to each consumer of type θ_i, a contract with quality q_i^* and price $\theta_i q_i^*$, capturing the full surplus S_i^* as his profits from this type of consumer.

We will refer to a product with quality q_ℓ^* and price $\theta_\ell q_\ell^*$ as *the first-best product for low valuation consumers*, and a product with quality q_h^* and price $\theta_h q_h^*$ as *the first-best product for high valuation consumers*.

3.2 Profit maximizing product line under full consideration: the Mussa-Rosen menu

We now turn to the profit maximizing problem under full-consideration and asymmetric information. The efficient product line above is no longer implementable as the monopolist cannot identify the valuation of each consumer directly. Instead, the monopolist must rely on a menu that screens consumers based on their preferences. Notice that offering the two products characterized above is not incentive compatible, as high valuation consumers will prefer to purchase a product of quality q_ℓ^* at price $\theta_\ell q_\ell^*$ rather than a product of quality q_h^* at price $\theta_h q_h^*$.

It is well known that the solution involves offering to high valuation consumers a product at the efficient quality level, $q_h^{mr} = q_h^*$, while distorting the quality provided to low valuation consumers, $q_\ell^{mr} < q_\ell^*$. In particular, the quality provided to low valuation consumers is implicitly defined by

$$\varphi'(q_\ell^{mr}) = \theta_\ell - \frac{\mu_h}{\mu_\ell}(\theta_h - \theta_\ell)$$

if the right hand side expression is positive, and $q_l^{mr} = 0$ if it is not the case. Prices are defined by the incentive compatibility constraint of the high type and the participation constraint of the low type respectively:

$$p_h^{mr} = \theta_h q_h^{mr} - (\theta_h - \theta_\ell) q_\ell^{mr}$$

$$p_\ell^{mr} = \theta_\ell q_\ell^{mr}.$$

Note that if $q_\ell^{mr} = 0$ then $p_h^{mr} = \theta_h q_h^{mr}$ which implies the seller captures all the surplus generated by high valuation consumers. Note that also in this case $p_\ell^{mr} = 0$, so low valuation consumers are receiving an offer with zero quality and price, obtaining zero utility as well. We interpret this zero-quality, zero-price product as low valuation consumers not being served in this market, i.e., they being excluded from the market. While in this case considering the product $(0, 0)$ an actual product or not makes no difference in terms of profits, it would make a difference in our model with partial consideration, as having them using some of the slots available reduces their availability for other products.

We will refer to the menu with $q_\ell^{mr} > 0$ as the *Mussa-Rosen menu with two products* and the menu with $q_\ell^{mr} = 0$ as the *Mussa-Rosen menu with a single product*.

4 The case of a single sample

We start with the case of a single sample: consumers can only observe one product from the menu. Note that this is the case in which frictions are extreme, and only the product in the first slot inspected by a consumer is the one that could be evaluated for purchase. We start this section discussing the performance of single-product menus in our setting, then we evaluate the performance of the Mussa-Rosen menu in this context, and explain why the optimal menu is a single-product menu in this case. Finally, we provide some comparative statics with respect to the fraction of high valuation consumers, discussing its effects on welfare and quality provision.

4.1 Single-product menus

We start discussing the performance of a simple class of menus in our setting: menus that contain a single product. In the standard setting without search frictions, this type of mechanisms are optimal only if the fraction of high valuation consumers is large enough. Otherwise, multi-product menus containing one product for each type of consumer typically dominate the performance of these single-product menus.

We now turn to the analysis of menus containing a single product.

In single-product menus a single combination of quality and price uses all available slots. Therefore, only this product will be inspected by all consumers regardless of the sample size they are endowed with.

Since we consider a setting with only two valuation levels, the profits obtained from a menu of this class will depend on whether the product is acceptable for all consumers or only high valuation consumers. If the product $(q, p, 1)$ is acceptable by all consumers, then the profits obtained by the monopolist are

$$p - \phi(q),$$

while if the product is acceptable only for high valuation consumers,

$$\mu_h(p - \phi(q)).$$

In order to be acceptable by all consumers, we must have

$$p \leq \theta_\ell q.$$

Since there is no gain on having this inequality to be strict, the monopolist will always set the price such that $p = \theta_\ell q$, conditional on her wanting to attend the whole market. If instead, she wants to exclude low valuation consumers, then the product must satisfy

$$p \leq \theta_h q,$$

and similarly, she will set $p = \theta_h q$ as there is no gain for the monopolist on setting a price below this level. Since in either case, the monopolist is capturing the whole surplus from offering a product of quality q, there is tradeoff between offering a higher quality and reducing the informational rents of the consumers as in the case in which there are more products. Then, by offering a product with quality q_i^*, the monopolist could guarantee to capture the maximum surplus conditional on the types of consumers she chooses to serve.

Proposition 1. *The best single-product menu is given by* $(q_i^*, \theta_i q_i^*, 1)$ *for some* $i \in \{\ell, h\}$.

The following assumption completely identifies the characteristics of the single product offered in this menu.

Assumption 1.

$$S_\ell^* > \mu_h S_h^*$$

Assumption 1 guarantees that offering only the first-best product for low valuation consumers is the best single-product menu that the monopolist could offer.

Corollary 1. *The best-single product menu is given by* $\{(q_\ell^*, \theta_\ell q_\ell^*, 1)\}$*, i.e., the first-best product for low valuation consumers.*

The assumption above will also play a key role in the characterization of the optimal menu when the sample size is $n = 2$ as we show below.

4.2 Mussa-Rosen menus with a single sample

We now turn to analyze the performance of the Mussa-Rosen menu when consumers inspect a single slot. Note that under Assumption 1, the Mussa-Rosen menu always contain two products. This means that the performance of the Mussa-Rosen under search frictions would be worse than the case without search frictions: any menu would involve imperfect matching between consumers and the products could purchase. The question remains on whether there are menus that could outperform it or not. We will show that in the case of a single sample this is the case and a simple menu with a single product would be optimal. When consumers inspect more than one slot, this is still true generically, but the distortions on quality provision are ambiguous.

For now, we focus in the case in which consumers can inspect only a single slot. Lets start with a menu containing only the products in the Mussa-Rosen menu with two products. Is it some version of this menu optimal in this case? The answer is negative: any menu with two or more products is dominated by a menu with a single product when consumers inspect a single slot.

By design, when a consumer inspect a slot with a product tailored to his valuation, he always decides to purchase it. If a high valuation consumer inspects a slot containing a product designed for the low valuation consumer he still purchases it since by design both products give him the same utility level. However, if a low valuation consumer inspects a slot with a product designed for the high valuation consumer he refuses to purchase from the monopolist. Then, due to this informational or matching friction, there is a chance that some consumers refuse to purchase from the monopolist despite all consumers always purchased from the monopolist under full-consideration (i.e., if they inspect all slots).

Consider first the case in which the fraction of high valuation consumers is very small (i.e., a value of μ_h close to zero). Note that under any menu containing both products in the Mussa-Rosen menu with two products, with positive probability a consumer inspects only the product intended for the high type, but this product is acceptable only by a very small fraction of consumers: the high valuation consumers.

The rest of the time, consumers evaluates a low quality product at a distorted quality level: the quality is below what would be efficient for low types. This product is acceptable for both types of consumers, and therefore purchased by any consumer inspecting a slot containing a product of this type.

Compare this menu with an alternative menu that contains only the first-best product for low valuation consumers: $(q_\ell^*, \theta_\ell q_\ell^*, 1)$. Note this product is also purchased by all consumers, and generates strictly larger profits from low types compared to the distorted offer in the Mussa-Rosen menu, but it generates lower profits from high valuation consumers that received a high quality offer before. For μ_h close to zero this last negative effect is negligible, so the positive effect dominates. This makes the original Mussa-Rosen menu suboptimal in this case.

As μ_h increases a high quality product becomes more profitable, but as long as Assumption 1 holds, including it in the menu would not exceed the profits obtained from offering only the first-best product for low valuation consumers if consumers inspect a single slot.

4.3 Optimal menu with a single sample

We have shown that the standard Mussa-Rosen menu is not optimal in general. The question that still remains is whether there are other "screening" menus with more than one offer that perform better in the case of a single sample. Our main result shows that this is not the case, and any screening menu is dominated by a menu with a single option.

There are two very simple steps that show that there is no better menu available for the monopolist. First, since each consumer will always observe a single product, there are no incentive compatibility constraints to consider when designing the structure of a particular product. Hence, only participation constraints will determine the form of each contract. This means that any product offered as part of an optimal menu must have a very simple form: its quality must match the efficient quality level for the lowest type willing to purchase such product, while its price is determined by the participation constraint of that specific type, therefore, extracting all the consumer surplus generated for that type. The second step involves showing that then the monopolist's problem could be written as a linear program over the products of this simple form, and that among those offers there will be one that generates higher profits for the monopolist.

Theorem 1. *Suppose $n = 1$. Then, the optimal menu contains a single product. Moreover, this product takes the form $(q^*_\ell, \theta_\ell q^*_\ell, 1)$.*

This implies that when search frictions are strong and consumers can only inspect a single slot, the incentives for the monopolist to offer a complete product line disappear, even in the cases in which a complete product line was offered in absence of search frictions. Then, the optimal menu is just to offer the best single-product menu: filling all slots with the first-best product for low valuation consumers (under Assumption 1).

4.4 Welfare effects and comparative statics

We provide some comparisons of the welfare effects for this case compared with the case of full-consideration (i.e., the case without search frictions).

Proposition 2. *Suppose $n = 1$. Then, the average quality provided is larger than under full-consideration.*

Search frictions have an impact on the average quality provided when $n = 1$: since the only product offered will have the efficient quality level for low valuation consumers, the average quality provided will typically increase. However, the quality of the products purchased does not increase for all consumers, as high valuation consumers end up with a product of lower quality compared to the case without search frictions.

Proposition 3. *Suppose $n = 1$. Then,*

- *Monopolist's expected profits are lower than under full-consideration.*

- *Expected consumer surplus is larger than under full-consideration.*

While high valuation consumers end up with a product of lower quality compared to the case without search frictions, they also pay a lower price for this product. The total effect over their utility is then positive, as the informational rents they would get under a differentiated menu are maximal when $q = q_\ell^*$. As the surplus for low valuation consumers is zero both in the case with or without search frictions, the expected consumer surplus is larger than in the case without search frictions.

Clearly, the monopolist is worse off as she could offer a single-product under full-consideration, but preferred to offer a differentiated menu in that case.

Proposition 4. *Suppose $n = 1$. Then,*

- *Expected consumer surplus and total welfare are increasing in μ_h.*
- *Monopolist's expected profits and quality provided are constant in μ_h.*

As the quality of the product offered remains constant in the fraction of high valuation consumers as long as Assumption 1 holds, the profits for the monopolist also remain the same. While the surplus of both types of consumers remain the same, an increase in the fraction of high valuation consumers increases the expected value of their surplus as high valuation consumers have a higher valuation for any quality level.

5 The case of two samples

We now consider the case in which consumers inspect two slots. Note that in this case consumers could end up inspecting two slots containing exactly the same product, reducing the effective size of their consideration sets.

As in the previous case, we will start discussing the performance of menus with a single product.

We will focus our analysis to the case in which the efficient low quality product is more profitable than the efficient high quality product, i.e., the case in which Assumption 1 holds. We will show below that under this assumption, single-product menus are never optimal.

We start introducing some notation. For a product i, let $u_h(i)$ and $u_\ell(i)$ denote the utility obtained by a low and high valuation consumer from purchasing i respectively.

For any menu M, we define L_M and H_M as the set of products accepted by all consumers and only high valuation consumers respectively. That is

$$L_M = \{i \in M : u_\ell(i) \geq 0\}, \textit{and}$$

$$H_M = \{i \in M : u_h(i) \geq 0 \text{ and } u_\ell(i) < 0\}.$$

We will refer to sets L_M and H_M as *classes* of products, so products in either L_M or H_M are products belonging to a different class of products. Note that any product in class L_M must have $p \leq \theta_\ell q$, while any product in H_M must have $\theta_\ell q < p \leq \theta_h q$.

Below we will describe some of the characteristics products in each class must satisfy if M is an optimal menu.

5.1 Single-product menus are not optimal

In the previous section, we show that the optimal menu with a single sample was to offer a single-product menu. The next result shows that an optimal menu must have more than one product if consumers could inspect more than one slot.

Proposition 5. *Suppose $n > 1$. An optimal menu contains at least two products, i.e., single-product menus are never optimal.*

The basic idea behind this result is that a menu that contains the first-best product for low valuation consumers and a product with quality $q = q_h^*$ and price $p = \theta_h q_h^* - (\theta_h - \theta_\ell) q_\ell^*$ could generate more profits that a menu with a single product, if slots are distributed appropriately across the two products. Under Assumption 1, such menu is guaranteed to exists: starting with a menu only with the first-best product for low valuation consumers, introducing a second product with the characteristics described before in a small proportion has two effects,

- Sometimes consumers inspect only this second product,
- Sometimes consumers inspect both products.

The first effect is negative under Assumption 1 as only high valuation consumers purchase this product, while the second effect is positive as the profits obtained by screening consumers with these products are larger than the profits obtained by selling the first-best product for low valuation consumers to everyone. Then, if the slots allocated to the second product are small enough, the second effect dominates as it is more likely that consumers end up inspecting slots containing both products instead of inspecting slots only containing the second product. Therefore, having a menu with at least two products is always better that having a single product.

This result also rules out the possibility of having only products of the same class: if consumers can only choose from products of a single class, either all from L_M or all from H_M, then it is better to offer a single product.

Corollary 2. *Suppose M is an optimal menu. If M contains only products of the same class, then $|M| = 1$.*

Having established that single-product menus are suboptimal, we now proceed to present some of the characteristics that an optimal menu must satisfy when consumers can inspect two slots.

5.2 Products quality

A standard characteristic of the optimal products in the Mussa-Rosen setting is that products targeted to high valuation consumers are provided at the efficient quality level for this type of consumers. In our setting with search frictions this characteristic of the optimal menu will remain unchanged under an additional assumption that rules out the possibility of over-provision of quality.[3]

Assumption 2. $\overline{Q} = q_h^*$.

Then, under this assumption, we can guarantee that in an optimal menu the products in class H_M are provided at the efficient quality level for high valuation consumers. That is, the standard *no distortion at the top* property of the optimal mechanism holds in our setting.

Lemma 1. *Consider an optimal menu M. Then, for any product $h \in H_M$, $q = q_h^*$.*

This result implies that any two products in class H_M can only differ in their price, but not their quality. This implies also that the there is a clear order among the products in this class for both the monopolist and the consumers: in this class of products, products

[3]This assumption is introduced for simplicity. We are confident this result is not required for all the results in this paper, but since in our setting we cannot rely on the revelation principle to pin down some of the characteristics of the menu, discarding overprovision of quality for the high quality products become more challenging.

with a higher price are preferred by the monopolist while products with a lower price are preferred by the consumers.

Products in different classes (L_M or H_M) will generate different levels of profits for the seller. However, under Assumption 1, any product in class H_M is associated to a larger profit than any product in class L_M when both of them are purchased (i.e., conditional on purchase).

Lemma 2. *Consider an optimal menu M. Suppose Assumption 1 holds. Then, for any $\ell \in L_M$ and any $h \in H_M$,*

$$p_h - \varphi(q_h) > p_\ell - \varphi(q_\ell).$$

That is, conditional on being sold, any product accepted only by high valuation consumers is strictly more profitable than a product accepted by all consumers.

If a product accepted by all consumers (i.e., a product of class L_M) is more profitable than some product accepted only by high valuation consumers (i.e., a product of class H_M), then by replacing this last product by the first-best product for low valuation consumers must increase profits as such product is the best single-product that could be offered.

We will use this result to show that in an optimal menu, the products in class L_M must not "over-provide" quality, that is the quality of any product accepted by all consumers must be below the efficient level for low valuation consumers. Note this property is also shared by the optimal menu without search frictions.

Another property of the optimal menu without search frictions is that low valuation consumers obtain zero utility from purchasing the product designed for them (and a negative payoff from the other product). We will show below that this also the case in our setting with search frictions. However, in this case both properties are tight together: it is not possible to show that there is no over-provision of quality in the low quality products and that low valuation consumers payoff is zero under an optimal menu. We show this in the result below.

Lemma 3. *Suppose M is an optimal menu. Then, for any product $\ell \in L_M$,*

1. *Low valuation consumers must obtain zero utility from product ℓ, and*

2. *$q_\ell \leq q_\ell^*$.*

The intuition behind this result is as follows: suppose initially that there is a single product ℓ in L_M and $q_\ell \leq q_\ell^*$. Then, if $u_\ell(\ell) > 0$, the monopolist could increase her profits by increasing the price of ℓ since she can collect a higher price from selling this product and at the same time makes product ℓ less attractive for high valuation consumers, increasing the chances they pick a product in class H_M which is more profitable as we shown in Lemma 2. If there are more than one product in L_M but their quality is still below q_ℓ^*, then for this argument to hold, we need to increase the prices all products at the same time, so that low valuation consumers are indifferent across all products in L_M. If there are products with quality above the efficient level for low valuation consumers ($q > q_\ell^*$), then the argument above still holds by combining all those products into a single one with quality $q = q_\ell^*$.

Note that Lemma 3 implies that all products in class L_M gives the same utility to low valuation consumers, extracting all the surplus generated by these types of consumers conditional on the quality they provide. Therefore, products in this class will differ in both their quality and price, but the later will be fully determined by their quality level. So, while products in class H_M are fully differentiated by their price, products in class L_M are identified by their quality.

Also note that products in class L_M will be ranked the same by both the monopolist and consumers: products with higher quality generate more profits for the monopolist and also more utility for the (high valuation) consumers.

5.3 Ordered products

The next step in our characterization is to establish an "order" over the products in an optimal menu that extends to products in both classes. For achieving this we will introduce a relation which we label *sorting relation* that ranks different products in the menu using two criteria: first according to the utility that high valuation consumers obtain from purchasing each product, and then breaking ties according to the product with the highest quality.

Definition 1. *We define the* sorting *relation $\succ$ over a menu M as follows: for any two products $i, j \in M$, $i \succ j$ if either (i) $u_h(i) > u_h(j)$, or (ii) $u_h(i) = u_h(j)$ and $q_i > q_j$.*

The sorting relation would allows us to order all products in a menu. We establish this formally in the result bellow.

Lemma 4. *Consider a menu M. M is a strictly totally ordered set according to the sorting relation $\succ$.*

Therefore, we can index the products in an optimal menu M according to $\succ$, so that $i+1 \succ i$ for all $i = 1, 2, ..., |M| - 1$. With this ordering in hand, we will be able to further characterize the structure of an optimal menu.

Note that a similar property is also present in the equilibrium characterization in Garrett et al. (2018). However, in their setting the ordering correspond to pair of products instead of individual products as consumers in their setting are always able to observe two different products that induce self-selection. Instead, in our setting search frictions affect the chances of consumers observing each product individually. Another difference is that **?** studies a competitive setting and search is between-firms, while we study a monopolist setting with within-firm search.

Lemma 5. *Suppose $n = 2$. Consider an optimal menu M indexed by the sorting relation $\succ$. For any $i = 1, 2, ..., |M| - 1$, either $i \in L_M$ and $(i+1) \in H_M$, or $i \in H_M$ and $(i+1) \in L_M$.*

In other words, no two products "next" to each other in terms of the high valuation consumers' payoffs can belong to the same "class": always one of them targets all consumers while the other targets only high valuation consumers. With this results in hand, we can show that the optimal menu in this case cannot be a balanced menu.

The intuition behind this result is straightforward: if there are two adjacent products of the same class, then it is better to combine them in a single product since this have no impact on the ranking with respect to other products according to the sorting relation. Therefore, if two adjacent products belong to class L_M, then by assigning all slots assigned to both products to the product with the highest quality among them, then profits of the monopolist increase. If instead both belong to class H_M, then better to allocate all slots involved to the product with the higher price.

The next two results shows that in an optimal menu products most products come in pairs: each product of class L_M has an associated product of class H_M that gives high valuation consumers the same utility level and vice-versa. That is, the "local" incentive compatibility constraint binds for each pair of products. The only product that could potentially break this pattern is the first-best products for high valuation consumers, since no product in L_M could be incentive compatible with it.

Lemma 6. *Consider an optimal menu M and a product $h \in H_M$. Then, if $u_h(h) > 0$, there must exists a product $\ell \in L_M$ such that*

$$u_h(h) = u_h(\ell).$$

Lemma 7. *Suppose $n = 2$ and consider an optimal menu M. For each $\ell \in L_M$, there exists a product $h \in H_M$ such that $u_h(\ell) = u_h(h)$.*

The argument behind these lemmas is as follows: suppose you start with two products ℓ and h of class L_M and H_M respectively, such that there are ranked next to each other according to the sorting relation. If $u_h(h) > u_h(\ell)$, then either a small increasing the price of h or a small increasing the quality and price of ℓ could be done without changing the ranking of any product in the menu. Therefore, such a change would increase the profits for the monopolist. Only if $u_h(h) = u_h(\ell)$ such a change is no longer possible.

This implies that for any product $h \in H$ such that $u_h(h) > 0$, there exists a product $\ell \in L_M$ such that $p_h = \theta_h q_h^* - (\theta_h - \theta_\ell) q_\ell$. This means that the quality of a product in class L_M also pin downs the price of a related product of class H_M. Since by definition a zero-quality zero-price product cannot be part of any menu, the only exception to this rule is for a product that provides zero-utility to high valuation consumers, that is, the first-best product for this group of consumers. We will show below that such product cannot be part of the menu, so indeed this rule characterize the price of any product of class H_M. Lemma 7 we can also reverse this relationship, and find a product of class H_M for any product in class L_M. Note that in this case the qualifier $u_h > 0$ is not required as high valuation consumers obtain a positive payoff from any product in class L_M.

5.4 No first-best products in an optimal menu

It should be no surprise that these results would allow us to determine that the number of products in an optimal menu must be even. But this requires to discard the first-best product for high valuation consumers as a potential member of the optimal menu.

For a given menu M and product $i \in M$, let $M_{\{-i\}}$ be the modified menu that excludes product i and redistributes the slots proportionally. That is,

$$M_{\{-i\}} = \left\{ (q, p, x) : (q, p) = (q_j, p_j) \text{ and } x = \frac{x_j}{1 - x_i} \text{ for some } j \in M \backslash \{-i\} \right\}.$$

The next two results establish that the first-best products cannot be part of an optimal menu in this case.

Lemma 8. *Suppose $n = 2$ and consider an optimal menu M. For any product $h \in H_M$, $p_h < \theta_h q_h^*$.*

Since for an optimal menu $q_h = q_h^*$ for any product $h \in H_M$, Lemma 8 implies that first-best product cannot be part of an optimal menu. Therefore, in an optimal menu, high valuation consumers must receive informational rents to incentivize them to pick a high quality product when such product is available to them. Due to the imperfect match between consumers and products, such informational rents cannot guarantee that high valuation consumers would pick up products in class H_M, as it is possible that some products in class L_M are still more attractive to the particular high quality product they have available.

Lemma 9. *Suppose $n = 2$ and consider an optimal menu M. For any product $\ell \in L_M$, $q_\ell < q_\ell^*$.*

The first-best product for low valuation consumers have quality $q = q_\ell^*$, therefore Lemma 9 rules out this product to be part of the optimal menu. So, while Lemma 3 already ruled out that there is over-provision of quality for products in class L_M, the previous lemma reinforces this results showing that in an optimal menu products in class L_M have strictly lower quality that the efficient level for low valuation consumers.

To understand why first-best products cannot be part of the optimal menu, fix a menu M and a product $i \in M$. Then, the monopolist's profits under menu M could be decomposed into three terms:

1. The profits obtained when both slots inspected containing this product,
2. the profits obtained when only one slot inspected contains this product,
3. the profits obtained when neither slot contains this product.

Suppose first the product i is the first-best product for high valuation consumer. Then, the profits in the first term are given by $\mu_h S_h^*$ as the monopolist collects the surplus S_h^* only from high valuation consumers. For the second term, since the utility obtained by a consumer picking this product is zero or lower, either consumers prefer another product if such product is available, or reject both products. Therefore, in this case the profits are determined entirely by the product in the other slot inspected. Finally, for the third term, this is equivalent to the profits obtained from a menu that excludes this product in the first place after adjusting the probabilities proportionally, i.e. menu $M_{\{-i\}}$.

Under Assumption 1, the first and second terms are bounded by the profits obtained from the first-best product for low valuation consumers, as this is the best single-product that could be offered to consumers. However, for the menu containing the first-best product for high valuation consumers to be optimal, it must be true that the profits it generates are larger than both the profits obtained from offering only the first-best for low valuation consumers and also the profits from offering a menu containing all products in M. But at the same time, the profits from the menu are bounded by a convex combination of the profits obtained from exactly those two menus. This leads to a contradiction.

Now consider the case in which product i is the first-best product for low valuation consumers and suppose initially that there is no product giving the same utility to high valuation consumers. Since the first-best product for low valuation consumers is the preferred product for all consumers, the first two terms the expected profits are simply the maximum surplus that could be obtained from low valuation consumers S_ℓ^*. Therefore, as before, the profits under menu M are a convex combination of S_ℓ^* and the profits obtained from menu $M_{\{-i\}}$. The optimality of menu M then leads again to a contradiction.

If there is a product h' giving the same utility to high valuation consumers, then this argument does not work as in some cases in the second term, the profits could be larger than both S_ℓ^* and the profits from menu $M_{\{-i\}}$. However, it could be shown that if that is the case, then the monopolist benefits from reducing the quality in product ℓ^* as this reduces the informational rents linked to product h'. Therefore, having the first-best product for low valuation consumers is also suboptimal in this case.

Having discarded the first-best products as part of an optimal menu, we can show that the number of products in an optimal menu must be even.

Lemma 10. *Suppose $n = 2$. Consider an optimal menu M. The number of products in M must be even.*

Recall that we have shown before that for any product in class H_M (with $u_h > 0$), we can find a product in class L_M such that high valuation consumers are indifferent, and therefore the products in an optimal menu will typically come in pairs. If the first-best products cannot be part of the optimal menu, then by the definition of a menu, there is no product i such that $u_h(i) = 0$. Therefore, for any product in M we can find a companion product of a different class such that both gave the high valuation consumers the same payoff.

5.5 Balanced and unbalanced menus

Our main result in this section establishes that in an optimal menu the monopolist always have some products that are more salient than others. That is, the distribution of slots is not uniform across products in an optimal menu.

A menu will be *balanced* if the fraction of slots used by all the products in the menu are the same. If some product is more salient than others, i.e., if $x_i > x_j$ for some i, then the menu will be *unbalanced*. In an unbalanced menu, products using more slots are more likely to be inspected, and therefore are more likely to enter consumers' consideration sets.

Definition 2. *We say menu M is balanced if $x_i = x_j$ for any $i, j \in M$ and $|M| > 1$. Otherwise, menu M is unbalanced.*

Note that under this definition, a single-product menu is an unbalanced menu. This is due to the interpretation that a menu with a single product is one in which the "bias" towards a product is so extreme that only that single product is ever inspected by consumers.

Increasing the number of slots allocated to a products increases the probability it enters consumers' consideration sets. However, due the limitation in the number of slots available, this means that also reduces the chances other products in the menu enter the consumers' consideration sets. Indeed this trade off between the salience of different products will influence the shape of the optimal menu.

Suppose a menu contains only two products: i and j. By our previous results, each one of them should be targeting different types of consumers (Corollary 2). Then, the profits obtained from this menu could be decomposed into three elements: the profits from consumers inspecting only product i, the profits from consumers inspecting only product j, and the profits from consumers inspecting both products. The probability of consumers inspecting slots containing at least one copy of each product is maximized when both products have the same salience. This also implies that the probability of having consumers inspecting slots only containing product i or only product j are also the same. However, if the profits from selling only one these products alone is larger than the profits from selling only the other product, then there is an incentive to "bias" the sampling towards such product, which results in an unbalanced menu. This incentive disappears only when both products generate exactly the same profits if sold in isolation, in which case it is optimal for the monopolist to offer a balanced menu.

Lemma 11. *Suppose M is an optimal menu with two products. If M is a balanced menu, then the expected profits of menus $\{(p_i, x_i, 1)\}$ and $\{(p_j, x_j, 1)\}$ must be the same for any $i, j \in M$.*

Therefore, an optimal menu could contain exactly two products only if both products generate the same profits. The next result shows that having both products generating the same profits cannot be optimal generically.

Proposition 6. *Suppose $n = 2$ and $\varphi(q) = \frac{q^\eta}{\eta}$ with $\eta \geq 2$. If there is an optimal menu M that contains only two products, then generically, menu M is unbalanced.*

The idea behind this result is that the optimality conditions that characterize the optimal menu with two products cannot be simultaneously satisfied by a balanced menu (generically) if the cost function takes the form $\varphi(q) = q^\eta/\eta$ for $\eta \geq 2$. Therefore, an optimal menu should either be unbalanced or contain more than two different products. However, in our main result below we show that even a balanced menu with more than two products cannot be optimal, so the optimal menu must always be unbalanced.

5.6 Main theorem

Theorem 2. *Suppose $n = 2$ and $\varphi(q) = \frac{q^\eta}{\eta}$ with $\eta \geq 2$. The optimal menu is unbalanced.*

Proof. Consider an optimal menu M. By Lemma 10, M must contain an even number of products. Let $|M| = 2N$ with $N \in \mathbb{N}$. By Lemma 6, $N > 1$. Let v_i be the profits obtained from selling product i and V_M be the expected profits obtained from menu M. Then,

$$V_M = \sum_{i=1}^{N} \frac{1}{(2N)^2} v_{2i-1} + \sum_{i=1}^{N} \frac{1}{(2N)^2} \mu_h v_{2i} + \sum_{i=1}^{N} \frac{2}{(2N)^2} \left(\mu_\ell v_{2i-1} + \mu_h v_{2i}\right) + \sum_{j=2}^{N}\sum_{i=1}^{j-1} \frac{2}{(2N)^2} v_{2j-1}$$

$$+ \sum_{j=2}^{N}\sum_{i=1}^{j-1} \frac{2}{(2N)^2} \mu_h v_{2j} + \sum_{j=2}^{N}\sum_{i=1}^{j-1} \frac{2}{(2N)^2} \left(\mu_\ell v_{2i-1} + \mu_h v_{2j}\right) + \sum_{j=2}^{N}\sum_{i=1}^{j-1} \frac{2}{(2N)^2} v_{2j-1}$$

Let $\overline{V}_{i,j}$ be the profits from a balanced menu containing only products i and j. Using that $\mu_\ell v_{2i-1} + \mu_h v_{2j} < \mu_\ell v_{2j-1} + \mu_h v_{2j}$ and $v_{2j-1} < \mu_\ell v_{2j-1} + \mu_h v_{2j}$ for $j > i$, and rearranging the terms in the expression above, we can obtain that

$$V_M < \frac{1}{N^2} \overline{V}_{1,2} + \sum_{j=2}^{N} \frac{2j-1}{N^2} \overline{V}_{2j-1,2j}.$$

Furthermore, since the right hand side of this expression is a convex combination of $V_{2j-1,2j}$ for $j = 1, ..., 2N-1$,

$$V_M < \max_{j=1,...,N} \overline{V}_{2j-1,2j}.$$

This contradicts M being optimal.

Therefore, a balanced menu could be optimal only if contains exactly two products. However, by Lemma 6, such a menu cannot be optimal either.

□

If the optimal menu M is balanced, then the profits under menu M could be bounded by the profits of a convex combination of a collection of balanced menus with only two products, each one containing an incentive compatible pair of products from the original menu M. This bound is strict unless M contains only two products. This means that choosing a menu containing only the most profitable pair of products under a balanced menu would be more profitable. However, by Lemma 6, an unbalanced menu would improve over the profits of any balanced menu with two products, so the optimal menu must be unbalanced.

Theorem 2 establishes that an optimal menu is always (up to a measure-zero set of parameters) unbalanced: even if it contains multiple different products, these products would have differences in their salience. In other words, the monopolist always has an incentive to "bias" sampling in favor of certain products, so that these products enter the consumers' consideration sets more often.

We summarize the main results of this and the previous section in the following corollary.

Corollary 3. *Suppose $n \leq 2$ and $\phi(q) = \frac{q^\eta}{\eta}$ with $\eta \geq 2$. Then, the optimal menu is unbalanced.*

5.7 Results for small menus

We know restrict to the case in which the maximum number of products is $\overline{m} = 3$. With this restriction, we can further characterize the optimal menu.

In particular, we can show that the optimal menu has exactly two products.

Corollary 4. *Suppose $n = 2$ and $\overline{m} = 3$. Then, the optimal menu contains exactly two products.*

Proof. Follows from Proposition 5, Lemma 10, and $|M| \leq 3$. □

Since the menu has only two products, each one of them should belong to a different class by Corollary 2. By our previous results then, it suffices to look to menus of the form

$$M(q_\ell, x_\ell) = (q_\ell, \theta_\ell q_\ell, x_\ell), (q_h^*, \theta_h q_h^* - (\theta_h - \theta_\ell)q_\ell, 1 - x_\ell).$$

The structure of each product in this menu come from our previous results. In particular, the price of the low quality product follows from Lemma 3, and the price of the high quality product follows from Lemma 7.

The profits under a menu $M(q_\ell, x_\ell)$ are given by

$$V(q_\ell, x_\ell) = (x_\ell^2 + 2x_\ell(1-x_\ell)\mu_\ell)S_\ell(q_\ell) + (1-x_\ell^2)\mu_h(S_h^* - (\theta_h - \theta_\ell)q_\ell).$$

Then, the optimal menu is characterized by the following two equations

$$\frac{x_h}{x_\ell} = \frac{\mu_h}{\mu_\ell}\frac{S_h^* - S_h(q_\ell)}{S_\ell(q_\ell)}, \text{ and}$$

$$\varphi'(q_\ell) = \theta_\ell - \frac{1-x_\ell^2}{x_\ell^2 + 2x_\ell x_h \mu_\ell}\mu_h(\theta_h - \theta_\ell).$$

These conditions correspond to the first order conditions of the monopolist's maximization problem, after restricting the number of products to two, and conditioning the structure using our previous results. Therefore, the optimal menu in this case can be fully characterized by two variables: q_ℓ and x_ℓ.

The first condition links the relative salience of each product to their relative profitability and the relative fraction of high and low valuation consumers. The numerator corresponds to the additional profits obtained from selling the high quality product instead of the low quality product, while the denominator are just the profits obtained by selling the low quality product.

The second condition, links the marginal cost of producing the low quality product of quality q_ℓ with its marginal benefit: the marginal value that could be capture from consumers θ_ℓ discounted by a term that depends on the informational rents that would be provided to high valuation consumers for having them picking the high quality product instead if available. This second term also depends on the relative probability of consumers picking the low quality product ℓ vs. the high quality product h, a measure of the relative importance of the informational rents vs. the profits obtained from consumers of the low quality products.

5.8 Welfare effects and comparative statics

The first comparison we can establish is in which cases the quality distortions introduced by the presence of asymmetric information are reinforced by the presence of search frictions.

Proposition 7. *Suppose $n = 2$ and $\overline{m} = 3$. Consider an optimal menu $M(q_\ell, x_\ell)$. Then, $q_\ell \leq q_\ell^{mr}$ only if $x_\ell < 1/2$.*

Note that the quality level of the low quality product will coincide with the quality in the Mussa-Rosen menu only if the first order conditions with respect to q_ℓ here and in the Mussa-Rosen menu coincide. That is, only if

$$\theta_\ell - \frac{1 - x_\ell^2}{x_\ell^2 + 2x_\ell x_h \mu_\ell}\mu_h(\theta_h - \theta_\ell) = \theta_\ell - \frac{\mu_h}{\mu_\ell}(\theta_h - \theta_\ell).$$

However, if $x_\ell \geq 1/2$ this condition never holds as the left hand side of this equation is guaranteed to larger in that case. In other words, in order for the quality to be equal or below the quality in the Mussa-Rosen menu, the monopolist should prefer to make the high quality product more salient.

Proposition 8. *Suppose $n = 2$ and $\overline{m} = 3$. Then,*

- *The quality provided in the low quality product could be larger, equal, or smaller than under full-consideration. Moreover, it is decreasing in μ_h.*
- *The average quality is lower than in the case in which consumers inspect a single slot (i.e., $n = 1$), even after conditioning on purchase.*

Proposition 9. *Suppose $n = 2$ and $\overline{m} = 3$. Then,*

- *Expected consumers surplus could be higher, equal, or lower than under full-consideration.*
- *Expected profits are lower than under full-consideration.*

Since the quality of the low quality product could be higher, equal, or lower in this case, the informational rents of high valuation consumers could also be higher, equal, or lower than in the case without search frictions. Since the monopolist has imperfect control over the matching between consumers and products, her profits are always lower than in the case with full-consideration. However, her=profts are always larger than in the case of a single sample: the monopolist can always offer a menu with a single product and obtains exactly the same profits as before.

For consumers, the opposite happens: while the comparison with the full-consideration set is ambiguous, they are always worse off compared to the case with a single sample. Recall that the high valuation consumers' informational rents are maximal at $q = q_\ell^*$, but for $n = 2$ we have $q < q_\ell^*$ for any product of class L_M in the optimal menu, and any product of class H_M generating the same payoff as some product of class L_M.

We now turn to discuss the effect of an increase in the fraction of high valuation consumers on the structure of the optimal menu.

Proposition 10. *Suppose $n = 2$ and $\overline{m} = 3$. The quality q_ℓ and salience x_ℓ of the low quality product in an optimal menu are decreasing in μ_h. This also implies,*

- *The price of the low quality product is decreasing in μ_h, and*
- *The price and salience of the high quality are increasing in μ_h.*

An increase in the fraction of high valuation consumers μ_h has two effects: first, it makes increasing the chances of having the high quality product sampled more desirable as it is more likely that a high valuation consumer ends up inspecting a slot containing this product. Second, it also increases the incentives for the monopolist to decrease the quality of the low quality product in order to reduce the informational rents of the high valuation consumers. Both effects reinforce each other as reducing the quality of the low quality product makes the high quality product more profitable and the low quality less profitable, which in turn makes allocating more slots to the high quality product more desirable.

6 Concluding Remarks

In this paper, we analyzed the product line design problem of a monopolist interacting with an unit-measure of consumers in presence of both asymmetric information and search frictions. In our setting, the monopolist designs a menu of products, which could differ both in quality and price, and must be positioned over a continuum of slots in order to be considerer by the consumers. Consumers inspect only a finite number of slots at random, and must decide whether to purchase on of the products in the inspected slots if any. We find that if consumers can only inspect a single slot, then the optimal menu for the monopolist includes a single product. Therefore, when distortions created by search frictions are severe, the monopolist prefers to shut down any differentiation in his product line, and focuses on offering a single product. If search frictions are less severe and consumers could inspect two slots, then the optimal menu takes a less extreme form and carries differentiated products. However, the presence of search frictions induce the monopolist to "bias" the allocation of slots such that some products enter the consumers' consideration sets more often than others. That is, the monopolist always have an incentive to offer a menu in which the distribution of slots is unbalanced.

The presence of search frictions in our setting makes the monopolist always worse off compared to the case with full-consideration. Therefore, the monopolist would have incentives to increase consumers' awareness and reduce search frictions if she is able to do so. Our results show that if search frictions could not be eliminated, or the matching process cannot be fully controlled by the monopolist, then she will adjust the product line in order to increase her expected profits, and this always result in a asymmetric menu: either she destroys any differentiation across products, or make some products more salient than others, despite having a balance menu could increase the chances consumers inspecting different products.

We focus on the problem of a monopolist in our model. Introducing competition would certainly have an impact on the types of products each firm will offer in equilibrium, and different competitive arrangements could different effects on the product line as well. We hope to explore the effects of competition in future research.

Our current model abstract from any form of direct targeting, as all consumers face the same menu and allocation of slots, and in consequence, have the same probability of evaluating each alternative. Since the use of advertising tailored to specific types of consumers is a common practice in current marketplaces, a natural extension of our model would consider the impact of personalized menus in the design problem of the firm.

Finally, while we focus on a standard product design problem of a firm in a vertically differentiated market, we think our model could be applied to more general settings in which the decisions of the agents are impacted by an imperfect match to a set of available alternatives, whose composition is in control of principal. For example, our framework could be adapted to analyze the problem in which potential beneficiaries of social benefits needs to interact with a complex set of alternative programs to apply for. Having these agents failing to consider all available programs they are eligible for could have consequences in terms of the design problem of a third party in control of these programs. Then, in light of our results, a simpler streamlined set of programs could be preferred if the search frictions are too severe, while for less extreme cases the advertisement or prioritization of the different programs would be impacted by the ultimate goal of the third party. We think our model could work as an initial framework to analyze this and other related program design decisions.

A Omitted proofs

A.1 Section 4

Proof of Proposition 1. Consider a menu with a single product $(q, p, 1)$. We need to consider two cases:

- If all consumers are willing to purchase product $(q, p, 1)$, we must have $p \leq \theta_\ell q$. Then, to maximize profits the monopolist sets $p = \theta_\ell q$, and her maximization problem could be written as

$$\max_q \theta_\ell q - \phi(q).$$

  This expression is maximized at $q = q_\ell^*$, which implies $p = \theta_\ell q_\ell^*$.

- If instead only high valuation consumers are willing to purchase product $(q, p, 1)$, we must have $p \leq \theta_h q$. Then, to maximize profits the monopolist sets $p = \theta_h q$, and her maximization problem could be written as

$$\max_q \mu_h(\theta_h q - \phi(q)).$$

  This expression is maximized at $q = q_h^*$, which implies $p = \theta_h q_h^*$.

□

Proof. Follows from Proposition 1 and Assumption 1. □

Proof of Theorem 1. Consider a product $(\hat{q}, \hat{p}, \hat{x})$ part of an optimal menu. Suppose $\hat{\theta}$ is the lowest accepting this offer but $\hat{\theta}\hat{q} - \hat{p} > 0$. Then, by increasing the price up to $\hat{\theta}\hat{q}$ this offer is still accepted by all types $\theta > \hat{\theta}$, and the incentives of all other offers remain the same. Hence, in order for $(\hat{q}, \hat{p})$ to be part of an optimal menu, it must be the case that $\hat{p} = \hat{\theta}\hat{q}$, where $\hat{\theta}$ is the lowest type accepting this offer. Then, any offer from an optimal menu must satisfy this property.

Again, as the structure of a particular offer has no influence on the incentives generated by all the other offers, it must be the case that the quality in an offer accepted by all types above θ_i maximizes

$$\theta_i q - \phi(q).$$

This is maximized at the efficient quality level for type θ_i, i.e., q_i^*. Hence, any offer part of an optimal menu must have quality q_i^* and price $\theta_i q_i^*$ for some i.

Then, in the case of two valuations, it suffices to compare the profits of a menu only containing $(q^*_\ell, \theta_\ell q^*_\ell, 1)$ and a menu containing only $(q^*_h, \theta_h q^*_h, 1)$ to determine which one is optimal. In the first case, all types of buyers accept the offer, generating profits equal to $\theta_\ell q^*_\ell - \phi(q^*_\ell)$, while in the later, only high valuation buyers accept the offer with associated profits $\mu_h \left(\theta_h q^*_h - \phi(q^*_h)\right)$. Comparing both expressions, we obtain that the first offer is strictly preferred if $\theta_\ell q^*_\ell - \phi(q^*_\ell) > \mu_h \left(\theta_h q^*_h - \phi(q^*_h)\right)$ and the second if $\theta_\ell q^*_\ell - \phi(q^*_\ell) < \mu_h \left(\theta_h q^*_h - \phi(q^*_h)\right)$. Finally, if $\theta_\ell q^*_\ell - \phi(q^*_\ell) = \mu_h \left(\theta_h q^*_h - \phi(q^*_h)\right)$, then the monopolist is indifferent between using any of the two offers, in any proportion, as both generate the same profits. The first case coincides with the condition in Assumption 1, which means offering only a product with quality q^*_ℓ at price $\theta_\ell q^*_\ell$ is optimal.

□

A.2 Section 5

A.2.1 Single-product menus are not optimal

Proof of Proposition 5. Suppose instead the optimal menu contains a single product. By Assumption 1, this product must be $(q^*_\ell, \theta_\ell q^*_\ell, 1)$, generating profits of S^*_ℓ with certainty as all consumers are willing to purchase this product and all slots contain the same product.

Consider the following menu: $M^*(x) = \{(q^*_\ell, \theta_\ell q^*_\ell, x), (q^*_h, \theta_h q^*_h - (\theta_h - \theta_\ell) q^*_\ell, 1 - x)\}$ for some $x \in (0, 1)$.

Let $V(x)$ be the profits under menu $M^*(x)$ as a function of x, and extend this function to $x = 1$ and $x = 0$ by setting $V(1) = S^*_\ell$ and $V(0) = \mu_h S^*_h$. The function V is strictly concave in x as long as $S^*_h > S_h(q^*_\ell)$, and maximized on an interior point $x^* \in (0, 1)$. This implies that the expected profits under the menu $M^*(x^*)$ are strictly larger than the profits under any menu containing a single-product. Therefore, menu M cannot be optimal. □

Proof of Corollary 2. If all products are of class L_M then, profits obtained from menu M are bounded above by S^*_ℓ since all consumers accept a product of this class and the maximum price that can be collected by the monopolist from this consumers coincide with the gross utility obtained by low valuation consumers. But, this coincides with the profits of offering only the first-best product for low valuation consumers. Therefore, it is better a menu containing only such product.

Similarly, if all products are of class H_M then, profits obtained from the menu are bounded above by $\mu_h S^*_h$ as only high valuation consumers accept this class of products, and at most the gross utility for high valuation consumers could be collected as the price.

But, offering a menu only containing the first-best product for high valuation consumers achieves exactly this level of profits, contradicting the optimality of the menu M. $\square$

A.2.2 Products quality

Proof of Lemma 1. Suppose not, that is, for an optimal M there is a product $h \in H_M$ such that $q_h < q_h^*$. Then, replacing h by $(q_h^*, p_h + \theta_h(q_h^* - q_h), x_h)$ doesn't change incentives for any type of consumer: low valuation consumers already rejected h, and this offer is even worse for them, while high valuation consumers receive the same utility from both offers. However, profits strictly increase since q_h^* maximizes the surplus generated by high valuation consumers.

Henceforth, in an optimal menu all products accepted only by high valuation consumers must have $q = q_h^*$.

$\square$

Proof of Lemma 2. Suppose not, that is, there is a product $\ell \in L_M$ such that the profits from selling ℓ are weakly larger than the profits from selling a product $h \in H_M$, i.e.,

$$p_\ell - \varphi(q_\ell) \geq p_h - \varphi(q_h)$$

Then, by replacing the quality and price of product h by the quality and price in the first-best product for low valuation consumers q_ℓ^* and $\theta_\ell q_\ell^*$, profits strictly increase since $S_\ell^* > S_\ell(q_\ell) \geq p_\ell - \varphi(q_\ell)$ for any $q_\ell \neq q_\ell^*$ and by Assumption 1 $S_\ell^* > \mu_h S_h^* \geq \mu_h S_h(q_h)$ for any q_h. Hence, M cannot be optimal. $\square$

Proof of Lemma 3. Suppose M is an optimal menu. First, let's assume $q_\ell \leq q_\ell^*$ for any $\ell \in L_M$. Consider an alternative menu M' that differs with M only in the price of the products in L_M. In particular, for any product $\ell \in L_M$, let the price in the menu M' be $p'_\ell = \theta_\ell q_\ell$. Clearly, now low valuation consumers obtain zero utility from any product in L_M with this modified price. Moreover, they are indifferent between any product in L and still reject any product in H_M. Therefore, their choices do not change while the profits obtained from them by the monopolist strictly increase as long as at least one these products has now a larger price.

For high valuation consumers, note that any product ℓ in $L_{M'}$ has become worse as their price has increased without changing its quality. Therefore, either preferences for the high valuation consumers will not change, or they will prefer a different product.

In the first case, the profits obtained from the monopolist will weakly increase. For the second case, we must consider two different situations:

1. First, when two products $\ell, \ell' \in L_{M'}$ are compared, a high valuation consumer will always prefer the product with the highest quality among them which is also the most profitable for the monopolist as $q_\ell, q_{\ell'} \leq q_\ell^*$.

2. Second, when a product $\ell \in L_{M'}$ is compared with a product $h \in H_M$, either a high valuation consumer will have the same preference as before, in which case the profits will weakly increase as the price of ℓ has increased, or he will prefer the product h over ℓ instead. By Lemma 2, product h must be more profitable than product ℓ at the original price. Henceforth, profits must increase for the monopolist in this case.

Since M' lead to higher profits, the original menu M cannot be optimal.

Now, suppose there are products in L_M with quality above q_ℓ^*. As before, consider an alternative menu M' such that it differs from M only in the price charged for products $\ell \in L_M$, so that the price under the menu M' is $p' = \theta_\ell q_\ell$, but also differs from M in that any product $\ell \in L_M$ such that $q_\ell > q_\ell^*$ is replaced by a product with quality $q'_\ell = q_\ell^*$. For a product $\ell \in M$, we will refer to the equivalent product in M' as the modified product ℓ. We just need to check that profits are increased when considering the products whose quality has been reduced. As in the previous case, low valuation consumers are indifferent among all products in $L_{M'}$ under the new menu M'. Note that the profits for the monopolist from selling a product $\ell \in L_{M'}$ at the modified price $p'_\ell = \theta_\ell q_\ell$ is equal to $\theta_\ell q_\ell - \varphi(q_\ell)$ which is increasing in q_ℓ for $q_\ell < q_\ell^*$ but decreasing in q_ℓ for $q_\ell > q_\ell^*$. Therefore, by replacing a product with $q_\ell > q_\ell^*$ by $q'_\ell = q_\ell^*$ the profits for the monopolist increase at the modified price.

We will show that the modified product can only be picked by a high valuation consumer if it was picked before. That is, if a product h was preferred to the original product then, h will also be preferred to the modified product. Note $\ell \in L_M$ implies that the original price $p_\ell \leq \theta_\ell q_\ell$.

A high valuation consumer obtains utility from the modified product

$$\theta_h q_\ell^* - \theta_\ell q_\ell^*$$

while he obtains from the original product

$$\theta_h q_\ell - p_\ell$$

Then, he will change his decision only if

$$\theta_h q_\ell^* - \theta_\ell q_\ell^* > \theta_h q_\ell - p_\ell$$

but $p_\ell \leq \theta_\ell q_\ell$ implies that this must satisfy

$$\theta_h q_\ell^* - \theta_\ell q_\ell > \theta_h q_\ell - p_\ell \geq \theta_h q_\ell - \theta_\ell q_\ell$$

we can rearrange the first and last expressions of this inequality as

$$\theta_h(q_\ell^* - q_\ell) > \theta_\ell(q_\ell^* - q_\ell),$$

but $q_\ell > q_\ell^*$ and $\theta_h \geq \theta_\ell > 0$ implies this condition never holds. Therefore, a modified product ℓ is choose by the high valuation consumers only in the cases in which the original product was choose under the menu M. This implies that monopolist profits increase as either it sells a product at a higher price, consumers decide to purchase a more profitable product, or sells a product generating a larger surplus which he captures.

Altogether, this implies that the profits for the monopolist increases under the modified menu M'. Therefore, the original menu M cannot be optimal, and an optimal menu must give zero utility to low valuation consumers and have $q \leq q_\ell^*$ for any product accepted by all consumers.

□

A.2.3 Ordered products

Proof of Lemma 4. By our definition of a menu, no two products in M can have the same quality and price. Therefore, the relation $\succ$ is complete in M. Transitivity and asymmetry follows from the transitivity and asymmetry of $>$ in $\mathbb{R}$. □

Proof of Lemma 5. Suppose products i and $i+1$ belong to the same class. Then,

- if both belong to L_M, replacing i's quality and price by $(i+1)'s$ quality and price strictly increases profits,

- if both belong to H_M, replacing $(i+1)'s$ quality and price by i's quality and price strictly increases profits.

Therefore, in an optimal menu M any succesive products must belong to different classes of products. $\square$

Proof of Lemma 6. Suppose not. That is, $u_h(h) > 0$, but there is no product $\ell \in L_M$ such that $u_h(h) = u_h(\ell)$.

We need to consider two cases. First, if there is no product $\ell \in L_M$ such that $u_h(\ell) < u_h(h)$, then increasing the price of h up to $\theta_h q_h^*$ has no effect on the ranking of the products by the consumers while expected profits for the monopolist strictly increase. Therefore, we must have $u_h(h) = 0$ if M is an optimal menu.

Now, suppose there is a product $\ell \in L_M$ such that $u_h(\ell) < u_h(h)$. By Lemma 5, this product must come just before h according to the sorting relation $\succ$. Then, increasing the price of product h has no impact on the ranking up to the point in which $u_h((q_h, p'_h, x_h)) = u_h(\ell)$, where p'_h is the price of h after the increase. Therefore, we must have $u_h(h) = u_h(\ell)$ in an optimal menu.

$\square$

Proof of Lemma 7. Consider an optimal menu M and a product $\ell \in L_M$. We need to consider two cases.

First, if there is a product $h \in H_M$ such that $u_h(h) > u_h(\ell)$, then by Lemma 5 this product must come next according to the sorting relation $\succ$. Then, increasing the price of product h has no impact on the ranking of products by the consumer up to the point in which the price makes high valuation consumers indifferent between ℓ and h, i.e., $u_h(h) = u_h(\ell)$.

If there is no such product, then if $q_\ell < q_\ell^*$ increasing the quality of product ℓ up to q_ℓ^* and the price up to $\theta_\ell q_\ell^*$ has no impact on the ranking of the products by consumers, and increases profits for the monopolist. However, this contradicts Lemma 9 as no such product could be part of an optimal menu.

$\square$

A.2.4 No first-best products in the optimal menu

Proof of Lemma 8. Suppose there is a product $h^* \in M$ such that $p_{h^*} = \theta_h q_h^*$. Let $V_{M'}$ be the expected profits obtained by the monopolist under menu M'. Then,

$$V_M = x_{h^*}^2 \mu_h S_h^* + 2x_{h^*}\left(\sum_{i \neq h^*} x_i \tilde{\mu}_i v_i\right) + (1 - x_{h^*})^2 V_{M_{\{-h^*\}}},$$

where v_i are profits obtained from selling the product i, and

$$\tilde{\mu}_i = \begin{cases} 1 & \text{if } i \in L_M \\ \mu_h & \text{if } i \in H_M \end{cases}$$

Under Assumption 1, $S_\ell^* \geq \tilde{\mu}_i v_i$ for any product $i \in M$, and the relation is strict for products in H_M. Therefore,

$$x_{h^*}^2 \mu_h S_h^* + 2x_{h^*}\left(\sum_{i \neq h^*} x_i \tilde{\mu}_i v_i\right) + (1 - x_{h^*})^2 V_{M_{\{-h^*\}}} < (1 - (1 - x_{h^*})^2) S_\ell^* + (1 - x_{h^*})^2 V_{M_{\{-h^*\}}}.$$

Since M is optimal, $V_M \geq V_{M_{\{-h^*\}}}$ and $V_M \geq S_\ell^*$. Combining this with the previous inequality, we obtain that

$$(1 - (1 - x_{h^*})^2) S_\ell^* + (1 - x_{h^*})^2 V_{M_{\{-h^*\}}} > S_\ell^*, \text{ and}$$

$$(1 - (1 - x_{h^*})^2) S_\ell^* + (1 - x_{h^*})^2 V_{M_{\{-h^*\}}} > V_{M_{\{-h^*\}}},$$

which leads to a contradiction. □

Proof of Lemma 9. Suppose there is a product $\ell^* \in M$ such that $q_{\ell^*} = q_\ell^*, p_{\ell^*} = \theta_{\ell^*} q_\ell^*$, and $x_{\ell^*} > 0$. Let V_M and $V_{M_{\{-\ell^*\}}}$ be the expected profits under menus M and $M_{\{\ell^*\}}$ respectively, and π_{ℓ^*} the probability product ℓ^* inspected along any other products (including ℓ^* itself).

We need to consider two cases: whether there is another product h^* in M such that $u_h(h) = u_h(\ell^*)$ or not. Consider first the case in which no such product exists. Then,

$$V_M = \pi_{\ell^*} S_\ell^* + (1 - \pi_{\ell^*}) V_{M_{\{-\ell^*\}}},$$

since the profits of selling ℓ^* are S_ℓ^* and ℓ^* is picked whenever ℓ^* inspected. Since M is an optimal menu, V_M must be weakly larger than the profits of selling just product ℓ^*, i.e.,

$V_M \geq S_\ell^*$. This implies that

$$\pi_{\ell^*} S_\ell^* + (1 - \pi_{\ell^*}) V_{M_{\{-\ell^*\}}} \geq S_\ell^*,$$

which implies

$$V_{M_{\{-\ell^*\}}} \geq S_\ell^*.$$

If $V_{M_{\{-\ell^*\}}} > S_\ell^*$, then offering menu $M_{\{-\ell^*\}}$ is strictly better for the monopolist.

If instead $V_{M_{\{-\ell^*\}}} = S_\ell^*$, then $V_M = S_\ell^*$. But, by Lemma 5, there exists a menu that generates strictly larger profits than S_ℓ^* for the monopolist.

In either case, the menu M cannot be optimal.

Now we analyze the case in which there is a product h^* such that $u_h(h^*) = u_h(\ell^*)$. By our definition of a menu, such product h^* must belong to class H_M. Moreover, we must have that $q_{h^*} = q_h^*$ and $p_{h^*} = \theta_h q_h^* - (\theta_h - \theta_\ell) q_\ell^*$. Consider the following menu $\tilde{M}(q) = M \backslash \{\ell^*\} \cup \{(q, \theta_\ell q, x_{\ell^*})\}$ for $q \in [\overline{q}, q_\ell^*]$,[4] where $\overline{q} = \sup_{i \in L_M \backslash \{\ell^*\}} q_i$.

Let q^* be the quality that maximize the expected profits from the menu $\tilde{M}(q)$. Then by maximizing the expected profits under menu $\tilde{M}(q)$ with respect to q, we obtain the following optimality condition for q^*:

$$\varphi'(q^*) \leq \theta_\ell - \frac{\pi_{h^*}}{\pi_{\ell^*}} (\theta_h - \theta_\ell),$$

where π_{h^*} and π_{ℓ^*} are the probabilities that products h^* and ℓ^* are purchased respectively. Note that $\varphi'(q_\ell^*) > \theta_\ell - \frac{\pi_{h^*}}{\pi_{\ell^*}} (\theta_h - \theta_\ell)$, as long as $\pi_{h^*} > 0$. Therefore, the profits under menu $\tilde{M}(q^*)$ must be larger than the expected profits under menu M. Thus, menu M cannot be optimal.

□

Proof of Lemma 10. Suppose not. That is, there is an optimal menu M with an odd number of products. Then, by Lemma 5, the lowest and highest products according to the sorting relation $\succ$ must belong to the same class. By Lemma 7, both products must belong to H_M since otherwise the number of products will be even. Denote them by $\overline{h}$ and $\underline{h}$. By Lemma 6, we must have that $u_h(\underline{h}) = 0$, otherwise the number of products will be even. But, by Lemma 1, such product must have $q_{\underline{h}} = q_h^*$. Finally, Lemma 8 implies that such a product cannot be part of an optimal menu.

□

[4] Here we are abusing notation by defining a menu with two products with quality $\overline{q}$.

A.2.5 Balanced and unbalanced menus

Proof of Lemma 11. Suppose an optimal menu M contains only two products a and b. By the definition of a menu, we have that $(q_a, p_a) \neq (q_b, p_b)$.

Let R_a and R_b, and R_{ab} be the expected profits obtained by the monopolist by consumers that observe only a, only b, and both a and b respectively. We assume without loss that $R_a \geq R_b$.

Suppose that contradicting the statement in the lemma above, M is balanced but $R_a > R_b$. Then, $x_a = x_b = \frac{1}{2}$. Note that M being optimal implies that $R_{ab} > R_a$ otherwise offering only product a would be optimal.

Consider the following menu indexed by x: $\tilde{M}(x) = \{(q_a, p_a, x), (q_b, p_b, 1-x)\}$. The expected profits under $\tilde{M}(x)$ could be written as

$$V(x) = x^n R_a + (1-x)^n R_b + (1 - x^n - (1-x)^n) R_{ab}.$$

In order to having a balanced menu as optimal, function V must be maximized at $x = \frac{1}{2}$. However, solving from the first order condition of maximizing $V(x)$ we have that

$$x^* = \frac{1}{1 + \left(\frac{R_{ab} - R_a}{R_{ab} - R_b}\right)^{\frac{1}{n-1}}}.$$

Then, if $R_a > R_b$, $x^* > \frac{1}{2}$, contradicting an optimal menu of two products being balanced.

□

Proof of Lemma 6. Consider an optimal menu M with only two products. Then, by Lemma 5, each product must belong to a different class. Let $\ell \in L_M$ and $h \in H_M$ be such products. The expected profits under menu M then are characterized by the following function

$$V(q_\ell, x_\ell, x_h) = x_\ell^2 S_\ell(q_\ell) + x_h^2 \mu_h (S_h^* - (\theta_h - \theta_\ell) q_\ell) + (1 - x_\ell^2 - x_h^2)(\mu_\ell S_\ell(q_\ell) + \mu_h (S_h^* - (\theta_h - \theta_\ell) q_\ell))$$

The first order conditions could be written as

$$\varphi'(q_\ell) = \theta_\ell - \left(\frac{1 - x_\ell^2}{x_\ell^2 + (1 - x_\ell^2 - x_h^2)\mu_\ell}\right) \mu_h (\theta_h - \theta_\ell), \text{ and}$$

$$\frac{x_h}{x_\ell} = \frac{\mu_h}{\mu_\ell} \left(\frac{S_h^* - S_h(q_\ell)}{S_\ell(q_\ell)}\right).$$

For a balanced menu to be optimal, both conditions must hold at $x_h = x_\ell = \frac{1}{2}$. This means, q_ℓ must satisfy simultaneously

$$\varphi'(q_\ell) = \theta_\ell - \left(\frac{3}{1+2\mu_\ell}\right)\mu_h(\theta_h - \theta_\ell), \text{ and}$$

$$\frac{\mu_h}{\mu_\ell}\left(\frac{S_h^* - S_h(q_\ell)}{S_\ell(q_\ell)}\right) = 1.$$

However, solving both equations simultaneously for q_ℓ is generically incompatible. □

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